PREFACE TO FIRST EDITION

The intended reader of this book is a graduate student beginning a doctoral program in physics or a closely related subject, who wants to understand the physical and mathematical foundations of analytical mechanics and the relation of classical mechanics to relativity and quantum theory.

The book's distinguishing feature is the introduction of extended Lagrangian and Hamiltonian methods that treat time as a *transformable coordinate,* rather than as the universal time parameter of traditional Newtonian physics. This extended theory is introduced in Part II, and is used for the more advanced topics such as covariant mechanics, Noether's theorem, canonical transformations, and Hamilton–Jacobi theory.

The obvious motivation for this extended approach is its consistency with special relativity. Since time is allowed to transform, the Lorentz transformation of special relativity becomes a canonical transformation. At the start of the twenty-first century, some hundred years after Einstein's 1905 papers, it is no longer acceptable to use the traditional definition of canonical transformation that excludes the Lorentz transformation. The book takes the position that special relativity is now a part of standard classical mechanics and should be treated integrally with the other, more traditional, topics. Chapters are included on special relativistic spacetime, fourvectors, and relativistic mechanics in fourvector notation. The extended Lagrangian and Hamiltonian methods are used to derive manifestly covariant forms of the Lagrange, Hamilton, and Hamilton–Jacobi equations.

In addition to its consistency with special relativity, the use of time as a coordinate has great value even in pre-relativistic physics. It could have been adopted in the nineteenth century, with mathematical elegance as the rationale. When an extended Lagrangian is used, the generalized energy theorem (sometimes called the Jacobi-integral theorem), becomes just another Lagrange equation. Noether's theorem, which normally requires a longer proof to deal with the intricacies of a varied time parameter, becomes a one-line corollary of Hamilton's principle. The use of extended phase space greatly simplifies the definition of canonical transformations. In the extended approach (but not in the traditional theory) a transformation is canonical if and only if it preserves the Hamilton equations. Canonical transformations can thus be characterized as the most general phase-space transformations under which the Hamilton equations are form invariant.

This is also a book for those who study analytical mechanics as a preliminary to a critical exploration of quantum mechanics. Comparisons to quantum mechanics appear throughout the text, and classical mechanics itself is presented in a way that will aid the reader in the study of quantum theory. A chapter is devoted to linear vector operators and dyadics, including a comparison to the bra-ket notation of quantum mechanics. Rotations are presented using an operator formalism similar to that used in quantum theory, and the definition of the Euler angles follows the quantum mechanical convention. The extended Hamiltonian theory with time as a coordinate is compared to Dirac's formalism of primary phase-space constraints. The chapter on relativistic mechanics shows how to use covariant Hamiltonian theory to write the Klein–Gordon and Dirac wave functions. The chapter on Hamilton–Jacobi theory includes a discussion of the closely related Bohm hidden variable model of quantum mechanics.

The reader is assumed to be familiar with ordinary three-dimensional vectors, and to have studied undergraduate mechanics and linear algebra. Familiarity with the notation of modern differential geometry is not assumed. In order to appreciate the advance that the differential-geometric notation represents, a student should first acquire the background knowledge that was taken for granted by those who created it. The present book is designed to take the reader up to the point at which the methods of differential geometry should properly be introduced—before launching into phase-space flow, chaotic motion, and other topics where a geometric language is essential.

Each chapter in the text ends with a set of exercises, some of which extend the material in the chapter. The book attempts to maintain a level of mathematical rigor sufficient to allow the reader to see clearly the assumptions being made and their possible limitations. To assist the reader, arguments in the main body of the text frequently refer to the mathematical appendices, collected in Part III, that summarize various theorems that are essential for mechanics. I have found that even the most talented students sometimes lack an adequate mathematical background, particularly in linear algebra and many-variable calculus. The mathematical appendices are designed to refresh the reader's memory on these topics, and to give pointers to other texts where more information may be found.

This book can be used in the first year of a doctoral physics program to provide a necessary bridge from undergraduate mechanics to advanced relativity and quantum theory. Unfortunately, such bridge courses are sometimes dropped from the curriculum and replaced by a brief classical review in the graduate quantum course. The risk of this is that students may learn the recipes of quantum mechanics but lack knowledge of its classical roots. This seems particularly unwise at the moment, since several of the current problems in theoretical physics—the development of quantum information technology, and the problem of quantizing the gravitational field, to name two—require a fundamental rethinking of the quantum-classical connection. Since progress in physics depends on researchers who understand the foundations of theories and not just the techniques of their application, it is hoped that this text may encourage the retention or restoration of introductory graduate analytical mechanics courses.

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